**Air Force Institute of Technology**

**Graduate School of Engineering and Management**

**Department of Electrical and Computer Engineering**

**CSCE 532 Automata and Formal Languages**

**Winter 2019**

# Day 2 – Finite Automata

§1.1 Finite automata

### Example (Sipser Exercise 1.1 )

The following is the state diagram of DFA . Answer the following questions about .

1. What is the start state?
2. What is the set of accept states?
3. What sequence of states does the machine go through on input aabb?
4. Does the machine accept the string aabb? No
5. Does the machine accept the string ? No

### Example (Sipser Exercise 1.2 )

Give the formal description of the machine pictured in Exercise 1.1

#### Solution

where is described by the following table

|  |  |  |
| --- | --- | --- |
|  | a | b |
|  |  |  |
|  |  |  |
|  |  |  |

### Example (Sipser Exercise 1.3)

The formal description of a DFA is , where is given by the following table. Give the state diagram of this machine.

|  |  |  |
| --- | --- | --- |
|  | u | d |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

#### Solution

### Practice (Sipser Exercise 1.1 )

The following is the state diagram of DFA . Answer the following questions about .

1. What is the start state?
2. What is the set of accept states?
3. What sequence of states does the machine go through on input aabb?
4. Does the machine accept the string aabb? Yes
5. Does the machine accept the string ? Yes

### Practice (Sipser Exercise 1.2 )

Give the formal description of the machine pictured in Exercise 1.1

#### Solution

where is described by the following table

|  |  |  |
| --- | --- | --- |
|  | a | b |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

### Example (Sipser Exercise 1.4a)

The language is the intersection of two simpler languages. Construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. .

#### Solution

Let and be the following DFAs, respectively.

Then accepts and accepts . The formal descriptions are and where , , , , , is

|  |  |  |
| --- | --- | --- |
|  | a | b |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

and is

|  |  |  |
| --- | --- | --- |
|  | a | b |
|  |  |  |
|  |  |  |
|  |  |  |

.

Let , , and . Then is

and accepts .

### Example (Sipser Exercise 1.14a)

Show that if is a DFA that recognizes language , swapping the accept and nonaccept states in yields a new DFA recognizing the complement of . Conclude that the class of regular languages is closed under complement.

#### Solution

Proof: Let and suppose, i.e. accepts . Then there exists a sequence of states such that

1. ,
2. , and
3. .

Now let and let be a sequence of states satisfying conditions 1 and 2. Because M and are deterministic, it follows by induction on that , and in particular that . Thus, does not accept , i.e. . On the other hand, if , then, so and so .

### Example (Sipser Exercise 1.5a)

The language is the complement of a simpler language. Construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language .

#### Solution

The following DFA recognizes .

Complementing the set of final (accepting) states yield the following DFA, which recognizes .

### Example (Sipser Exercise 1.6a)

Give the state diagram of a DFA recognizing .

#### Solution

### Practice (Sipser Exercise 1.4b)

The language is the intersection of two simpler languages. Construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. .

#### Solution

Let and be the following DFAs, respectively.

Then accepts and accepts . The formal descriptions are and where , , , , , is

|  |  |  |
| --- | --- | --- |
|  | a | b |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

and is

|  |  |  |
| --- | --- | --- |
|  | a | b |
|  |  |  |
|  |  |  |
|  |  |  |

.

Let , , and . Then is

and accepts .

### Practice (Sipser Exercise 1.5b)

The language is the complement of a simpler language. Construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language .

#### Solution

The following DFA recognizes.

Complementing the set of final (accepting) states yields the following DFA, which recognizes .

### Practice (Sipser Exercise 1.6c)

Give the state diagram of a DFA recognizing .

#### Solution

### Example (Sipser Exercise 1.23)

Let be any language over the alphabet . Prove that iff .

#### Solution

Proof: We must show that . However, by definition of , we have , so we only have to show . Also, the forward direction follows immediately from .

It remains only to show that that given , every string in is in . Let . Then where and every . Suppose . Then . Now suppose that for some it is the case that every and consider where each . By the inductive hypothesis, , so , which completes the proof.